

Example 2: Scale - up

Consider the scale-up of a fermentation from 10 L to 10000 L vessel. The small fermenter has a height-to-diameter ratio of 3. The impeller diameter is 30% of the tank diameter. Agitator speed is 500 rpm and three Rushton impellers are used.

Determine the dimensions of the large fermenter and agitator speed for:

- a) Constant P/V
- b) Constant impeller tip speed
- c) Constant Reynold number

Assume geometric similarity

$$\text{Help: } V = (\pi/4) D_T^2 H$$

Solution: Assume the vessel is a cylinder

Thus

$$V = \left(\frac{\pi}{4}\right) \cdot D_T^2 H$$

but $H = 3 D_T$ (as aspect ratio is 3:1)

So

$$V = \left(\frac{\pi}{4}\right) \cdot 3 \cdot D_T^3 = 10000 \text{ cm}^3 (= 10 \text{ L})$$

Solving for D_T , H and $D_{\text{impeller}} \Rightarrow (D_i = 0.3 \cdot D_T)$ gives

$$D_T = 16.2 \text{ cm}$$

$$H = 48.6 \text{ cm}$$

$$D_i = 4.9 \text{ cm}$$

The scale-up factor is the cube root of ratio of tank volumes, or

$$\left(\frac{10000 \text{ L}}{10 \text{ L}} \right)^{1/3} = 10$$

To maintain geometric similarity the larger vessel will have its dimension increased by a factor 10

That is: $D_T = 1.62 \text{ m}$

$$H = 4.86 \text{ m}$$

$$D_i = 0.49 \text{ m}$$

Now for

a) $P/V \sim \text{const.} \Rightarrow$

For constant P/V to apply $N^3 \cdot D_T^2$ must be the same in both vessels. Let's subscript 1 refer to the small vessel and 2 to the larger vessel

That means:

$$N_1^3 D_{T1}^2 \approx N_2^3 D_{T2}^2$$

$$\Rightarrow N_2 = N_1 \left(\frac{D_{T1}}{D_{T2}} \right)^{2/3} = 500 \text{ rpm} \left(\frac{1}{10} \right)^{2/3} = 107 \text{ rpm}$$

b) Tip speed $\approx$ constant $\Rightarrow$

$$N_1 D_{T1} = N_2 D_{T2}$$

$$\Rightarrow N_2 = N_1 \left(\frac{D_{T1}}{D_{T2}} \right) = 500 \cdot \left(\frac{1}{10} \right) = 50 \text{ rpm}$$

c) $Re \sim \text{constant}$

$$N_1 D_{T1}^2 = N_2 D_{T2}^2$$

$$\Rightarrow N_2 = N_1 \left(\frac{D_{T1}}{D_{T2}} \right)^2 = 500 \text{ rpm} \cdot \left(\frac{1}{10} \right)^2 = 5 \text{ rpm}$$

Therefore: Scale-up on the basis of constant P/V would be the most likely choice unless culture was unusually shear sensitive. Scale-up on constant Re would not be used